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Comparison of linear and nonlinear Control Strategies for the Stabilization of Rotary Pendulum

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***Abstract* - This paper deals with the comparative study between linear and nonlinear control strategies applied on a highly nonlinear, under actuated, unstable, non-minimum phase, Rotary Pendulum system, to stabilize its position on an unstable equilibrium point. The linear control technique such as linear quadratic regulator along with nonlinear control technique, sliding mode controller are implemented and compared on the linearized and nonlinear model of the system respectively. The simulations are carried out in MATLAB.**

***Index Terms*—Rotary inverted pendulum, Degrees of freedom, Linear quadratic regulator, Sliding mode control, Linearization**

I . INTRODUCTION

Control of Rotary pendulum is a challenging problem in the area of control engineering due to its easily developed dynamics and complexity of controller design. It is used for verifying the performances and demonstrating the effectiveness of various control algorithm techniques. The RIP (Rotary inverted pendulum) is a two degree of freedom system i.e., having two independent closed loop transfer functions. The RIP system has a L shaped arm which is connected to a shaft of a dc motor which pivots between ± 180 º. A pendulum is suspended on a horizontal axis at the end of the arm. The control variable is the control input that is given to the dc motor and the output variables are the angle of the pendulum and the angle of the shaft. The inverted pendulum is a classic experiment used to teach dynamics and control systems. The pendulum dynamics are derived using Lagrangian equations and an introduction to nonlinear control is made. There are three control challenges for the rotary pendulum system: designing a stabilization/balance controller designing a swing-up control and a switching mechanism which intercepts pendulum when it nears the upright position and switches to

stabilizing control. This paper deals with the design of a stabilization controller using various classical and modern

linear controlling techniques on a linearized model of the rotary pendulum system, to make the system stabilize around its unstable equilibrium position . Many methods are proposed for achieving stabilization of this system in literature. The recent papers relating the rotary pendulum includes all types of controllers, such as fuzzy control, sliding mode control, LQR, PID, MPC etc. The paper [8]

describes about the nonlinear control law and related tasks relevant for control theory courses. It deals with the modeling of nonlinear systems and discussion on selecting suitable solvers ODE Solvers.Stabilization is done using PD based linear and nonlinear state feedback techniques by eliminating small disturbances .In [5] a technique called optimal linearization which points out the disadvantages with the conventional Jacobian linearization technique ,for minimal approximation error is used. The proposed linear model is valid for any operating point including off equilibrium points. In [9] a hybrid control structure with advantages of sliding mode control and fuzzy control is considered and implemented, a strategy that is based on error criterion that suppress chattering and gives faster response which is compared with a single sliding mode control. In [11] a double closed-loop control scheme wasproposed for a cart-pendulum setup. The inner loop regulated the angle of pendulum by using root locus control, while the outer loop controlled the position of the car by using fuzzy logic control method.The paper is organized in the following fashion. Section IIbriefly describes the mathematical modelingof the rotary pendulum system derived using the Euler-Lagrangian principles. Section III describes about the control strategies thatare implemented on the linearized model as well as nonlinear of the rotary pendulum to attain stability. Section IV deals with the comparative simulation resultanalysis of the various control strategies applied on the Rotary Pendulum system and the studies are concluded in Section V.

II. MATHEMATICAL MODELING

When approaching the solution of a control problem, mathematical modeling is the basis of many of modern control strategies. The more that is known about a dynamic system, the more accurate a mathematical model can be. Accurate mathematical modeling allows the design of faster, more accurate and effective controllers. This is because mathematical models allow design, test and development of controllers rapidly using MATLAB even before the physical rig is constructed.Accurate mathematical models of the pendulum system can be developed provided that there is not excessive stiction (static friction), backlash or bearing slack present. Therefore mentioned non linearity’s make accurate modeling and control of the system much more difficult.

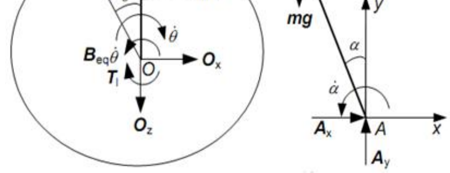

Figure 1 Free body diagram of arm and pendulum

Figure 1 shows the free body diagram of arm and pendulum of a rotary inverted pendulum respectively where the arm rotates about the Y axis and its angle is denoted by the symbol θ while the pendulum attached to the arm rotates about its pivot and its angle is called α. The shaft of the DC motor is connected to the arm pivot and the input voltage of the motor is the control variable.

Assumption is made that the pendulum angle, α, is defined to be positive whenit rotates counter-clockwise. That is, as the arm moves in the positive clockwise direction, the inverted pendulum moves clockwise (i.e. the suspendedpendulum moves counter-clockwise) and that is defined as α>0. Lagrangian equations are used to derive the dynamics between the pendulum angle α, arm angle Θ and the motor torque applied to the arm pivot T_{output} [2].Potential energy of arm is 0.So the total potential energy of the system is given by potential energy of the pendulum.

$$PE = PE_{pend} = mgh = mgL\cos\alpha \quad (1)$$

The Kinetic energy of arm is given by:

$$KE_{arm} = KE_{hub} + KE\, V_x + KE\, V_y \quad (2)$$

Total kinetic energy of the system,

$$KE_{tot} = + KE_{pendulum} + KE_{arm} \quad (3)$$

$$KE_{tot} = \frac{1}{2} * J_{eq}\dot{\theta}^2 + \frac{1}{2} * m\left(r\dot{\theta} - L\cos\alpha\,\dot{\alpha}\right)^2 + \frac{1}{2} * m(-L\sin\alpha\,\dot{\alpha})^2 + \frac{1}{2} * J_{eq}\dot{\alpha}^2$$

$$= \frac{1}{2}\left(J_{eq} + mr^2\right)\dot{\theta}^2 + \frac{2}{3}mL^2\dot{\alpha}^2 - mLr\cos\alpha\,\dot{\theta}\dot{\alpha} \quad (4)$$

After expanding the equation and collecting terms, the Lagrangian can be formulated as,

$$L = KE_{tot} - PE_{tot} \quad (5)$$

The two generalized co-ordinates are Θ and α. So, another two equations are:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \emptyset} = Toutput - Beq\dot{\theta} \quad (6)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\propto}}\right) - \frac{\partial L}{\partial \propto} = 0 \quad (7)$$

We get,

$$-mLr\cos\alpha\,\ddot{\theta} + \left(\frac{4}{3}\right)mL^2\ddot{\alpha} - mgL\sin\alpha = 0 \quad (8)$$

$$\left(J_{eq} + mr^2\right)\ddot{\theta} - mLr\cos\alpha\,\ddot{\alpha} + mLr\sin\alpha\,\dot{\alpha}^2 = T_l - B_{eq}\dot{\theta} \quad (9)$$

Solving these equations and linearizing about anequilibrium point α=0, by Taylor series expansion, and eliminating higher order terms, i.e., sinα=α, cosΘ=1

Symbol	Description	Values
m	Mass of Pendulum Arm	0.128 Kg
B_{eq}	Equivalent viscous damping coefficient	5 X e^{-3}
η_g	Gearbox efficiency	0.9
η_m	Motor efficiency	0.69
J_{eq}	Equivalent moment of inertia at the load	2 X e^{-3} Kg/m^2
K_g	system gear ratio	70
K_m	Back EMF Constant	7.67 X e^{-3}
K_t	Motor Torque Constant	7.67 X e^{-3}
R_m	Armature Resistance	2.6Ω
r	Rotating Arm Length	0.2 m
L	Length to Pendulum's Center of Mass	0.175m

$$-mLr\ddot{\theta} + \left(\frac{4}{3}\right)mL^2\ddot{\alpha} - mgL\alpha = 0 \quad (10)$$

$$\left(J_{eq} + mr^2\right)\ddot{\theta} - mLr\ddot{\alpha} = T_{output} - B_{eq}\dot{\theta} \quad (11)$$

The output Torque of the motor which act on the load is defined as,

$$T_l = \eta_g\eta_m K_t K_g/R_m\,(V - K_g K_m\dot{\theta}) \quad (12)$$

TABLE I
DYNAMIC MODEL PARAMETER

Finally, by combining the above equations, the following state-space representation of the complete system is obtained.

$$\begin{bmatrix}\dot{\emptyset}\\ \dot{\propto}\\ \ddot{\emptyset}\\ \ddot{\propto}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & bd/E & -cG/E & 0\\ 0 & ad/E & -bG/E & 0\end{bmatrix}\begin{bmatrix}\emptyset\\ \propto\\ \dot{\emptyset}\\ \dot{\propto}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ c.\eta g.\eta m.kt.kg/Rm.E\\ b.\eta g.\eta m.kt.kg/Rm.E\end{bmatrix} V \quad (13)$$

where,

$a = J_{eq} + mr^2$
$c = 4/3\ mL^2$
$G = (\eta_g.\eta_m.K_t.K_m.K_g^2 + B_{eq})/R_m$
$b = mLr$
$d = mgL$
$E = ac - b^2$

The above matrix equation (13) can be decoupled in terms of θ , α , $\dot{\theta}$, *and* $\dot{\propto}$

$$\ddot{\theta} = bd/E\alpha - cG/E\dot{\theta} + c.\eta.\eta m.kt.kg/Rm.EVm \quad (14)$$

$$\ddot{\alpha} = ad/E\alpha - bG/E\dot{\theta} + b.\eta g.\eta m.kt.kg/Rm.EVm \quad (15)$$

By applying Laplace transform,thetransfer functions G(α,θ) and G(θ,α) are found.
To validate the model the values are substituted on (14) and (15) as shown in Table I.

III. CONTROL STRATEGIES

To verify a modern control theory the inverted pendulum control can be considered as a very good example in control engineering. The inverted pendulum is highly nonlinear and open-loop unstable system that makes control more challenging. It is an intriguing subject from the control point of view due to its intrinsic nonlinearity. Common control approaches such as 2 DOF Transfer function based design, FSF control and LQR requires a good knowledge of the system and accurate tuning in order to obtain desired performances. However, an accurate mathematical model of the process is often extremely complex to describe using differential equations. Moreover, application of these control techniques to a humanoid platform, more than one stage system, may result a very critical design of control parameters and stabilization difficulty.
The problem of balancing an inverted pendulum is like balancing a vertical stick with your hand by moving it back and forth. Thus by supplying the appropriate linear force, the stick can be kept more-or-less vertical. In this case, the pendulum is being balanced by applying torque to the arm. The balance controller supplies a motor voltage that applies a torque to the pendulum pivot and the amount of voltage supplied depends on the angular position and speed of both the arm and the pendulum.

B. LINEAR QUADRATIC REGULATOR

The Linear Quadratic Regulator(LQR) is a type of optimal control strategy, in whichone attempts to find a controller that provides the best possible performancewith respect to some given measure of performance. E.g., the controller that uses theleast amount of control-signal energy to take the output to zero. In this case the measureof performance (also called the optimality criterion) would be the control-signal energy. Thus LQR is used to regulate the system about its upright equilibrium point. As the name may suggest the LQR controller requires a linear system for which it will generate constant gains for full state feedback to make the equilibrium point globally asymptotically stable. However the dynamics of inverted pendulum systems are inherently nonlinear. This leaves the problem of how to implement a control methodology designed for a linear system on a nonlinear system. The chosen approach was to linearize the equations of motion about the operating point and define a domain of attraction within which the constant gain controller results in local asymptotic stability. Delivering the system to the domain of attraction was achieved by a different method. Within the realms of MATLAB a full state feedback LQR controller is developed by solving the Algebraic Ricatti Equation based upon an effort weighting matrix and a state penalty matrix. For this the nonlinear dynamical equations must be written in the linear state space format(13). Further, it should be noted that the resulting LQR regulates only about a zero equilibrium. Since the equations of motion are not zero about the desired operating point and that in general the upright equilibrium can be described by an infinite number of coordinates, some kind of filtering of the signal passed to the constant gain controller is needed.

The system based on the dynamics of Rotary pendulum is found to be unstable. So to enhance the stability of the system a LQR controller is used.

The basic principles of LQR linear quadratic optimal control, by the system equation is

$$\dot{X} = Ax + Bu \quad (16)$$
$$Y = Cx$$

And the quadratic performance index functions

$$J = \tfrac{1}{2}\int (x'Q\,x + u'R\,u)\ dt \quad (17)$$

Q is a positive semi-definite matrix, R is positive definite matrix. If this system is disturbed and offset the zero state, the control u can make the system come back to zero state and J is minimal at the same time. Here, the control value u is called optimal control the control signal should be:

$$u = -Kx \quad (18)$$
where, $K = R^{-1} B' P$ (19)

Where P(t) is the solution of Riccati equation, K is the linear optimal feedback matrix. The next step is to solve the Riccati equation (21) to find the value of K:

$$A'P - P A + Q - PB R^{-1} B' P = 0 \quad (20)$$

Using the LQR method, the effect of optimal control depends on the selection of weighting matrices Q and R, if Q and R are not selected properly, the solution cannot meet the actual system performance requirements. In general, Q and R are taken as diagonal matrices, the current approach for selecting weighting matrices Q and R is using bryson rule[10],after finding a suitable Q and R, it allows the use ofcomputers to find the optimal gain matrix K easily.

C. SLIDING MODE CONTROL

Sliding mode control is an important robust control approach. For the class of systems to which it applies, sliding mode controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision. On the other hand, by allowing the tradeoffs between modeling and performance to be quantified in a simple fashion, it can illuminate the whole design process. The design of this control can be divided into three major very dependent steps, these latter include[4]:

- Choice of surface
- Establishment of conditions for the existence of convergence
- Determination of the control law.

Nonlinear systems can be described in canonical form as follows:

$$x^{(n)} = F(x) + G(x)u \quad (21)$$

$$y = x \quad (22)$$

State space representation of the rotary inverted pendulum is as follows,

$$\dot{x_1} = x_2 \quad (23)$$

$$\dot{x_2} = F1(x) + G1(x)u \quad (24)$$

$$\dot{x_3} = x_4 \quad (25)$$

$$\dot{x_4} = F2(x) + G2(x)u \quad (26)$$

Where,

$$F1 = \frac{-\frac{b^2 \sin(2x_1) x_2{}^2}{2} - Gb x_4 \cos(x_1) + adsin(x_1)}{ac - b^2 \cos(x_1)} \quad (27)$$

$$G1 = \frac{\eta_m \eta_g K_t K_g b \cos(x_1)}{R_m (ac - b^2 \cos(x_1))} \quad (28)$$

$$F2 = \frac{cF1(x) - dsin(x_1)}{b\cos(x_1)} \quad (29)$$

$$G2 = \frac{\eta_m \eta_g K_t K_g c}{R_m (ac - b^2 \cos(x_1))} \quad (30)$$

Considering the zero dynamics of the system, two sliding surfaces should be defined as,

$$s_\alpha = c_1 x_1 + x_2 \quad (31)$$

$$s_\theta = c_2 x_3 + x_4 \quad (32)$$

$$S = s_\alpha + c_3 s_\theta \quad (33)$$

Here c_3 is a positive variable. The effect of that is to put more stress on the control of the inverted pendulum rather than the control and stability of the motor.
To guarantee the stability of the feedback system, we drive the control signal u such that,
$\dot{S} = 0$we can get the control and the resulting control law is,

$$\dot{S} = -k_1 \text{sign}(S) \quad (34)$$

As a result the control law will be,

$$u = \frac{-(F1 + c_1 x_2 + c_3 F2 + c_2 c_3 x_4 + k_1 sign(S))}{G1 + c_3 G_2} \quad (35)$$

The behavior on the sliding mode depends only on the switching surface and is independent of the structural properties. Therefore the effectiveness of control is insensitive to parametric uncertainties of the model.

For reducing the chattering function we can use saturation function.

$$\text{sat}\left(\frac{\text{S}}{\text{phi}}\right) = \begin{cases} \frac{\text{S}}{\text{phi}} & \text{if phi} < abs(S) \\ \text{sign(s)} & \text{otherwise} \end{cases} \quad (36)$$

Resulting control signal is given by,

u =

$$= \frac{-(alpha * sat\left(\frac{S}{phi}\right) + F1 + c_1 x_2 + c_3 F2 + c_2 c_3 x_4 + k_1 sign(S))}{G1 + c_3 G_2} \quad (37)$$

IV. COMPARITIVE RESULT ANALYSIS

Figure 13 shows the step response of the linearized model of the rotary pendulum system with LQR controller. It shows the response of both arm angle and the pendulum angle. The weights given to the Q and R matrices determine the response of the LQR controller. As we vary these matrices the response of the system can be increased (settling time, rise time) along with overcoming the overshoot conditions In general, Q and R are taken as diagonal matrix, such that Q is positive semi definite and R is positive definite matrix The simplest way of choosing Q and R is by taking Q=C'C and R=1.

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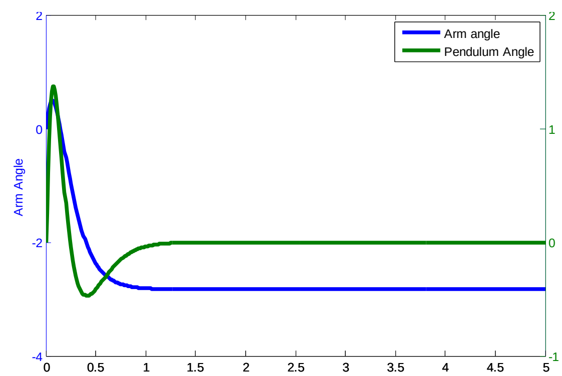

Figure 2 Step Response of RIP System with LQR

Figure 3 shows the SMC response of the pendulum angle of the rotary pendulum.The voltage applied to the plant is in the applicable range. In addition, the position of the inverted pendulum is regulated to its zero position, which is assumed to be on the top. Control of the inverted pendulum is much more important than the control of the position of the motor. The gain c_2 helps to have more emphasis on the control of the inverted pendulum than the control of the motor. As it is shown in figure theinverted pendulum approaches its zero position and then the position of the motor is controlled easily.

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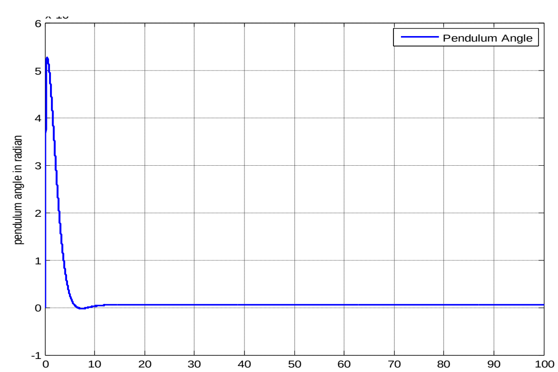

Figure 3 SMC response of pendulum angle

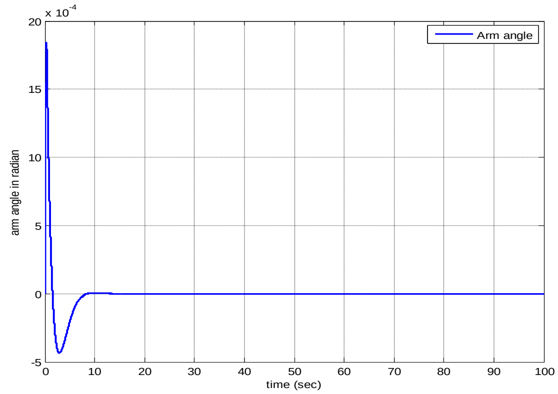

Figure 4 SMC response of arm angle

TABLE 2

COMPARISON AMONG DIFFERENT CONTROLLERS IMPLEMENTED

	LQR	SMC
Settling Time (sec)	2.5	5
Max Deviation From the desired Value	.011	.00052

V. CONCLUSION

Designing sliding mode controller for an under actuated system is difficult in general. Here, first the desired performance is introduced and based on this performance two sliding surfaces are designed, then system is controlled by proper definition of a lyapunov function. The lyapunov function designed puts more emphasis on the control of the inverted pendulum rather than the control of the motor.Hence a comparative study is done between the nonlinear and linear controlling techniques, where robust performances is shown by both .As the LQR technique is used over a linearized model, it might lose some of the best properties of the system while linearizing. SMC overcomes this disadvantage, by using the nonlinear model.

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