

Analysis of a Single Input Z-Source DC-DC Converter

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Abstract—This paper is related to investigate characteristics and performance of a single input Z-source dc/dc converter which boosts the input voltage to a higher output voltage. Z-source structure increases the reliability of the converter. Operating principles of the Z-source dc-dc converter is described by current and voltage waveforms of the components and mathematical expressions. Waveforms obtained and mathematical expressions are confirmed by simulations.

Keywords—Z-source, Single input Z source dc/dc converter, CCM

I. INTRODUCTION

This Z-source structure can be used in all types of power conversions such as ac/dc rectifiers, dc/dc converters, dc/ac inverters and ac/ac converters. The main operation principles of Z-source structure and application of Z-source structure to inverter are investigated in detail in [1]. The study discusses the drawbacks of traditional voltage-fed and current-fed inverters also. In voltage-fed inverter, the output ac voltage can not exceed the input dc voltage and upper and lower switches of the same phase can not be made 'ON' at the same time during the operation. Furthermore, in the current-fed inverter, the output ac voltage is always greater than the input dc voltage, and one of the upper switches and one of the lower switches must be 'ON' at any time during the operation. Z-source inverter eliminates these problems and gives an opportunity of using the inverter as a step-up or step-down inverter. The single phase ac/ac converter application of Z-source structure is investigated in [2]. The most popular topology for the ac/ac converter for the requirement of different output voltage level and variable output frequency is the usage of cascaded diode rectifier and inverter, respectively. However, if only the voltage regulation at the output side is of concern, the single phase Z-source ac/ac converter can provide a cheaper and lower-sized solution, [2]. Also, the proposed single-phase Z-source ac/ac converter in [2] can be used to tackle voltage sags, surges and load fluctuations.

Moreover, the operating principle of the three-phase Z-source ac/ac converter is investigated in [3]. The application of Z-source structure in single-phase rectifier is proposed in [4]. Also, the operating principle of the proposed circuit is

investigated in that study. Using single phase Z-source rectifier instead of traditional two-stage ac/dc buck rectifier brings some advantages. It gives the opportunity to adjust the output dc voltage greater or smaller than the input ac voltage. Also, the minimized and single stage structure of the proposed rectifier provides high efficiency and small size, [4]. Furthermore, the three phase rectifier application of Z-source structure is proposed in [5]. In that study, the power-factor of the rectifier is discussed together with the operating principle. For dc/dc application of Z-source structure, the main studies are [6] and [7].

The objectives of this paper are to present (1) the equivalent circuits and the associated expressions corresponding to different stages of operation of the PWM Z-source dc-dc converter in CCM, (2) the dc input-to-output voltage conversion factor for single input Z-source dc-dc converter.

Section II presents the equivalent circuits and the derivation of relevant equations of the PWM Z-source dc-dc converter [8]. Section III presents an analysis of the Z-source dc-dc converter in CCM mode and presents the derivation of DC voltage conversion factor for CCM. Section IV presents conclusions.

II. CIRCUIT CONFIGURATION

A. Z-Source Converters

Z-source converters are modern group of power electronic converters which can overcome problems with traditional converters. The Z-source converter is a novel topology [1] that overcomes the conceptual and theoretical barriers and limitations of the traditional voltage-source converter and current-source converter. The concept of Z-source was used in direct ac-ac power conversion [2]. Similarly, the concept of source also was extended to dc-dc power conversion [8].

B. Circuit Configuration of Proposed Converter

The schematic circuit diagram of the proposed single input Z-source dc-dc converter is shown in Fig. 1. It consists of one input source, V_s , and the diode, D_1 , applied to provide

current path . In this paper, permanent connection of input dc source is considered. Energy receiver, converter and transmitter sections are situated in the middle side of the converter. This section is a two-port network that consists of a split-inductor L_{z1} and L_{z2} and capacitors C_{z1} and C_{z2} connected in x-shape which is named “Z-network”. An active switch, Q , is situated in output port of Z-network to control input and output power of converter. The final section of converter is a LC filter beside the load in order to reject output signal ripple.

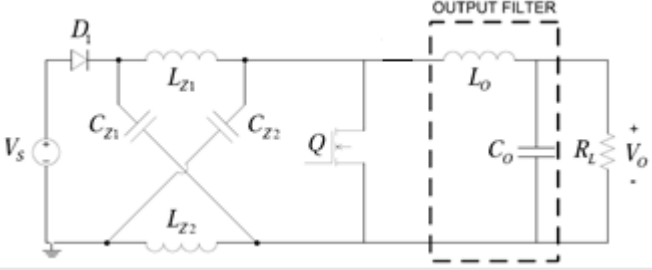


Fig.1. Single Input Z source dc-dc Converter

III. ANALYSIS OF SINGLE INPUT Z SOURCE DC-DC CONVERTER

The analysis of the input voltage to output voltage equation in terms of the dutyfactor, D , and other circuit components (inductors, capacitors, load resistance) is made for CCM operation. To use the symmetrical behavior of Z-source structure, the Z-source capacitors (C_{z1} and C_{z2}) are set equal to each other and Z-source inductors, (L_{z1} and L_{z2}) are chosen such as their sizes are same. Then, by the symmetry, voltage waveforms on Z-source inductors come out identical. The current waveforms through Z-source capacitors are also identical over a period. Dc component and small signal components in Z-source capacitor currents are same which is proven at [6]. This fact is same for the inductor voltages as well. So, if

$$\begin{aligned} L_{z1} &= L_{z2} = L_z \\ C_{z1} &= C_{z2} = C_z \end{aligned} \quad (1)$$

A. Mathematical Analysis of Z-source DC/DC Converter in CCM Operation

The circuit diagram of simplified Z-source dc/dc converter can be represented as in Fig.2.

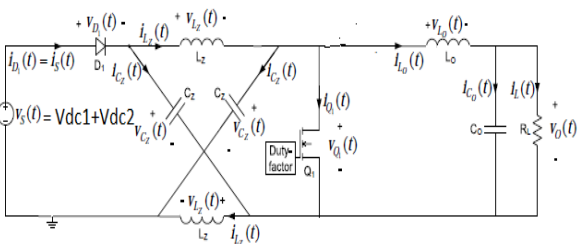


Fig.2. Circuit diagram of the Z-source dc/dc converter

In CCM operation, operation of Z-source dc/dc converter in one period can be divided into two modes. Mode 1 begins when switch, Q , is switched on at $t=0$. During this time interval t_1 appearing in Fig.6, the Z-source inductors, L_z are energized by Z-source capacitors, C_z . If Kirchhoff's voltage law is applied around LOOP II in Fig.3, it can be resulted that Z-source capacitor voltage, $v_{Cz}(t)$, is equal to Z-source inductor voltage, $v_{Lz}(t)$, at time interval t_1 . Also, using Kirchhoff's voltage law around LOOP I in Fig.3, gives an expression for D_1 voltage, $v_{D1}(t)$, is equal to $v_s(t) - 2v_{Cz}(t)$. As $v_{Cz}(t)$ is equal to output voltage, $v_o(t)$, and $v_s(t) < v_o(t)$ because of boosting operation, $v_{D1}(t)$ takes negative value. D_1 is reverse biased and does not permit current flow towards source. The load meanwhile is fed by the output inductor, L_o , and output capacitor, C_o . Also, output inductor voltage, $v_{L_o}(t)$, is equal to $-v_o(t)$ according to LOOP III in Fig.3. The equivalent circuit for Mode 1 is represented in Fig.3.

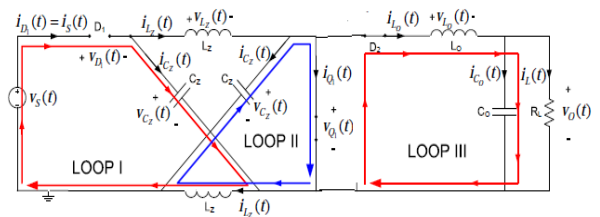


Fig.3 Equivalent circuit for Mode 1 in continuous current mode (CCM) operation of the Z-source dc/dc converter

Calling the time elapsed in one cyclic operation of the converter as period, T_s , Mode 2 starts at the instant dT_s when Q switched off at that instant. During the time period in Mode 2, Z-source inductors, L_z , transfer the stored energies on them to the load. Also, the current drawn from the input is transferred to Z-source capacitors C_z and load. Inductor L_o is energized during Mode 2. If Kirchhoff's voltage law is applied around LOOP I in Fig.4, it can be obtained that the output inductor voltage, $v_{L_o}(t)$, is equal to $2v_{Cz}(t) - v_s(t) - v_o(t)$ in Mode 2 operation. Similarly, applying Kirchhoff's voltage law at LOOP II brings that Z-source inductor voltage, $v_{Lz}(t)$, is equal to $v_s(t) - v_{Cz}(t)$ in Mode 2. The equivalent circuit for Mode 2 is shown in Fig.4.

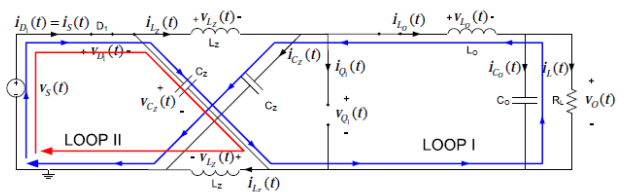


Fig.4 Equivalent circuit for Mode 2 in continuous current mode (CCM) operation of the Z-source dc/dc converter

If the capacitor sizes are chosen large enough, the voltage variation across the capacitors over a period is very small in steady state. Also, the input voltage can be determined as constant over a period. Thus, the voltages on capacitors and the input voltage are only dc.

$$\begin{aligned} V_{C_Z}(t) &= V_{C_Z} \\ V_{C_O}(t) &= V_{C_O} \\ sV_S(t) &= V_S \end{aligned} \quad (2)$$

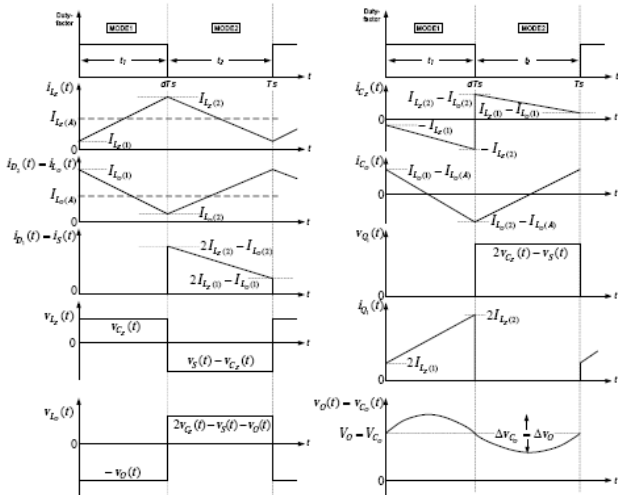


Fig.5 Voltage and current waveforms of the inductors (L_Z and L_O), the capacitors (C_Z and C_O) and the nonlinear elements (D_1 and Q) of the Z-source dc/dc converter in CCM operation

The voltages on inductors can be expressed in terms of capacitor voltages and input voltage at any time of period. As the capacitor and input voltages are dc, the voltages on the inductors are dc at any time of the period so, the slope of inductor current is constant and the current rises and falls linearly at the inductors. If the capacitor sizes are small, then the voltages across them become time dependent. As the voltages of capacitors are time dependent, then the voltages on the inductors become time dependent and this leads to complexity in derivations.

The voltage induced in an inductor due to a current, passing through it, is given by,

$$e_L = L \frac{di}{dt} \quad (3)$$

The voltage and current waveforms of inductors, capacitors and nonlinear elements of the circuit for continuous conduction mode are shown in Fig.5. In the figure $I_{LZ(A)}$, $I_{LZ(1)}$ and $I_{LZ(2)}$ are the abbreviations of average, valley and peak values of Z-source inductors current, respectively. Similarly, $I_{LO(A)}$, $I_{LO(1)}$ and $I_{LO(2)}$ are the abbreviations of average, peak and valley values of output inductor current. The voltages appearing on inductors, L_Z , are equal to V_{C_Z} in Mode 1 operation, thus considering the differential form of (3) expressed in incremental forms of variables then it can be derived;

$$V_{C_Z} = L_Z \frac{I_{LZ(2)} - I_{LZ(1)}}{dT_S} \quad (4)$$

Or

$$dT_S V_{C_Z} = L_Z (I_{LZ(2)} - I_{LZ(1)}) \quad (5)$$

whose left hand side represents volts-second area developed on L_Z inductors during Mode 1 operation. 'd' represents the duty-factor in the equations. Similarly, the voltage on the Z-source inductors, L_Z , are $V_S - V_{C_Z}$ in Mode 2.

B.DC Voltage Conversion Factor for CCM

$$(1 - d) (V_S - V_{C_Z}) = L_Z (I_{LZ(1)} - I_{LZ(2)}) \quad (6)$$

Incremental form of (3) yields; (6)

whose left hand side represents volts-second area developed on L_Z inductors during Mode 2 operation. Volt-second areas developed on inductors, L_Z , in one complete switching period, T_S , is to be zero. Using this fact, relationship can be obtained between V_S and V_{C_Z} . Sum (5) and (6) side by side for the purpose so that result will be;

$$V_{C_Z} = \frac{(1-d)V_S}{1-2d} \quad (7)$$

Applying the same approach to L_O yields another equation in terms of V_S , V_O and V_{C_Z} . The voltage developed on inductor, $v_{L_O}(t)$, at time interval t_1 is

$$V_{L_O}(t_1) = -V_O \quad (8)$$

Also, in time interval t_2 ,

$$V_{L_O}(t_2) = 2V_{C_Z} - V_S - V_O \quad (9)$$

Since the volt-second area of output inductor, L_O , in one switching cycle is to be zero, again adding (8) and (9) side by side yields;

$$dT_S(-V_O) + (1 - d).T_S.(2V_{C_Z} - V_S - V_O) = 0 \quad (10)$$

Substituting V_{C_Z} , obtained in (7), into (10)

$$\text{gives; } (dT_S(-V_O)) + \left[(1 - d).T_S \left[2 \frac{(1-d)V_S}{1-2d} - V_S - V_O \right] \right] = 0 \quad (11)$$

and which when simplified yields;

$$V_O = \frac{(1-d)}{1-2d} V_S \quad (12)$$

That is the dc input-to-output voltage conversion factor as

$$\frac{V_O}{V_S} = \frac{(1-d)}{(1-2d)} \quad (13)$$

IV. SIMULATION ANALYSIS

The main requirements of the Z-source dc/dc converter are listed at Table 1. According to these requirements Z-source inductors, L_Z , and output inductor, L_O , values are determined and designed Z-source dc/dc converter is simulated.

Table 1 Parameters of the Z-source dc/dc converter in CCM

Minimum input voltage	V_S	30	V
Output voltage	V_O	60	V
Output power	P_O	360	W
Peak-to-peak ripple current in Z-source inductor at nominal input voltage (in % of I_{LZ})	i_{LZD}	83.3	%
Peak-to-peak ripple current in output inductor at nominal input voltage (in % of I_{LO})	i_{LOD}	66.6	%

Table 2 Converter Parameters

Z-source inductors, L_Z	20	μH
Z-source capacitors, C_Z	50	μF
Output inductor, L_O	50	μH
Output capacitor, C_O	400	μF
Switching frequency, f_s	100	kHz
Load resistance, R_L	10	Ω
Input voltage, V_S	30	V
Output voltage, V_O	60	V
Output current	6	A

Fig.6 represents the power stage of the converter. The voltage source, V_S , corresponds to input voltage which is set to 30V. Duty-factor block generates the required duty-factor and its value is set to 0.333, to get 60V output voltage across the load resistance, R_L . The Z-source inductors, L_{Z1} and L_{Z2} , are set to 20mH, and the Z-source capacitors C_{Z1} and C_{Z2} , are chosen as 50mF. The output inductance, L_O , is assigned to 50mH. Also, the output capacitor, C_O , is chosen as 400mF. The forward voltage drops on diodes, D_1 and D_2 , are taken as zero, because the forward voltages of the diodes have not been taken into account in development of the converter model. R_L is set to 10W to draw 360W power from the supply, V_S .

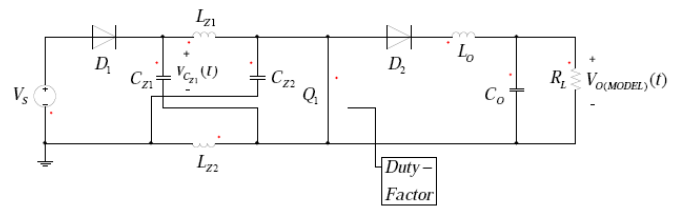


Fig.6. Power stage of the Z-source dc/dc converter in CCM operation

The results obtained in the simulations are shown in figures starting with Fig.7

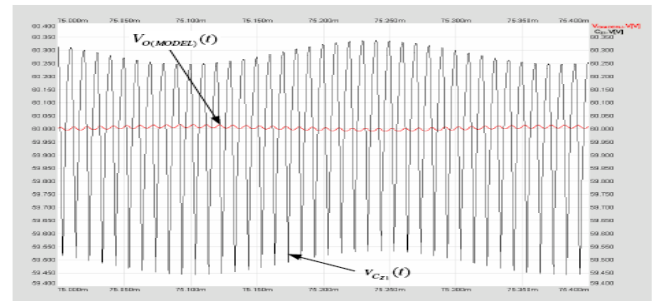


Fig.7. Voltage across the load resistor R_L , $V_{O(MODEL)}(t)$, (red), voltage across the Z-source capacitor C_{Z1} , $v_{CZ1}(t)$, (black) in CCM operation

The graphs display responses beginning from 75ms on word in order to discard the start up transients. The simulation results seen Fig.7 shows that; i. the output voltage, $V_{O(MODEL)}(t)$, on R_L is at 60V as desired; ii. and the voltages on Z-source capacitors, $v_{CZ1}(t)$, are equal to the output voltage, $V_{O(MODEL)}(t) = 60$, as the converter operation requires it. Fig.8 shows the following variables; Z-source inductor current, $i(t)$, Z-source capacitor current, $i_{CZ}(t)$, and diode D_1 current, $i_{D1}(t)$ graphically for CCM operation. Note that, in Fig.8 that when duty-factor output (in red) is high i.e. the switch Q_1 is 'ON', Z-source capacitors, C_{Z1} and C_{Z2} feed current (and hence energy) (black for $i_{LZ}(t)$ and green for $i_{CZ}(t)$) through the Z-source inductors, L_{Z1} and L_{Z2} , respectively. In this time interval, t_1 , D_1 is reverse biased and in blocking state, so no energy will be delivered to the rest of the converter by the source. When the duty-factor output falls to low at zero volts, i.e. the switch is 'OFF' the time interval t_2 is entered. In t_2 , Z-source inductors transfer the energy they have stored in t_1 to the load and the output inductor, L_O . D_1 is now forward biased and Z-source capacitors start charging (green for $i_{CZ}(t)$) from the input source during t_2 .

The peak-to-peak magnitude of the current through Z-source inductors is expected to be 10A as design criteria. Note that, in Fig.8 the inductor current, $i_{LZ}(t)$, swings between 7A and 17A in comply with the design criteria. Note also that, the average currents through L_{Z1} and L_{Z2} are 12A.

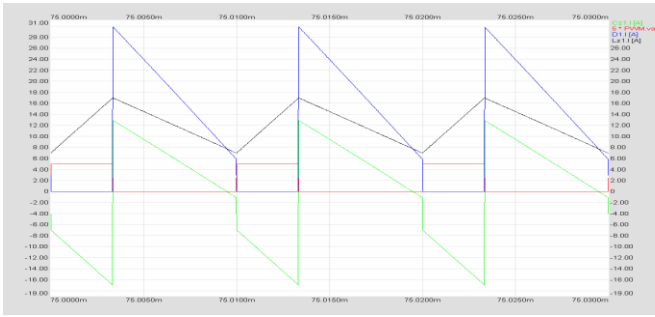


Fig.8 Z-source capacitor current, $i_{CZ}(t)$, (green), Duty-factor (red), diode D_1 current, $i_{D1}(t)$, (blue), and Z-source inductor current, $i_{LZ}(t)$, (black) in CCM operation

For CCM operation, the output inductor current, $i_{LO}(t)$, the output capacitor current, $i_{CO}(t)$ and the Z-source inductor current, $i_{LZ}(t)$, waveforms are displayed in Fig.9 together with duty-factor waveform. When the duty-factor output becomes high, at the beginning of time interval 1 t time interval $i(t)$ LO starts decreasing. This means, output inductor, L_O , transfers the stored energy in its magnetic medium to the output capacitor, C_O , and to the load, R_L .

When the Duty-factor output becomes low, i.e. the switch becomes 'OFF' in time interval t_2 , L_O starts storing energy over inductors L_{Z1} and L_{Z2} . The load is fed by C_O in the first half part of the time interval, t_2 . Note that the peak-to-peak current ripple on the L_O current is 4A due to the inductor current swing from 4A to 8A. Thus, the average value of L_O current is 6A.

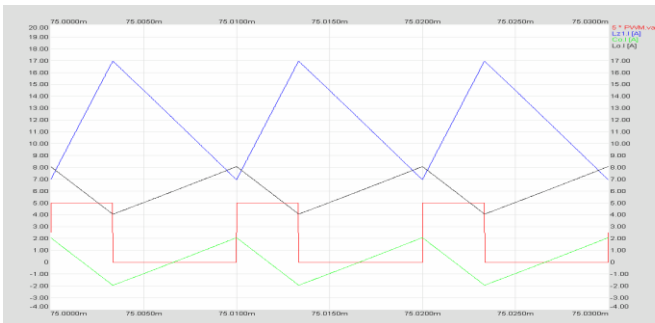


Fig.9 Duty-factor (red), Z-source inductor current, $i_{LZ}(t)$, (blue), output capacitor current, $i_{CO}(t)$, (green), and output inductor current, $i_{LO}(t)$, (black) in CCM operation

Waveforms seen in Fig.8. and Fig.9. are same with those theoretically expected shown in Fig.5. Also, the peak-to-peak ripple inductor currents are same with those theoretically calculated. Furthermore, in simulation, taking the duty-factor, $D = 0.333$, leads to 60V output voltage as calculated in (3.3). Thus, Z-source dc/dc converter simulations for the CCM operation support theoretical results.

V. CONCLUSION

A detailed steady-state analysis of single input Z-source dc-dc converter operating in CCM has been presented. The analysed waveforms are drawn according to the analysed equations. The dc input-to-output voltage conversion factor for an ideal single input Z-source dc-dc converter has been derived. Analysis is proven with the help of simulation.

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