A NOVEL IDEA BASED ON SWIRL MOTION OF WATER FOR FUNCTION OPTIMIZATION

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Abstract*— This paper proposes a new optimization algorithm based on vortex particle theory. Here particles represent multiple solutions that swirl (rotate) around a centre point called vortex. Vortex particle theory considers the fitness value of current position of particles that is obtained by applying an updating equation as well as strength update on previous position of particles. Since the proposed modifications make the vortex location in to a more steady position thus it widens the exploration capability of the algorithm. The performance of the proposed algorithm is tested using six bench mark functions. From the simulation result it is found the exploration ability of the proposed algorithm is not constrained through a narrow zone and produces better results without getting struck in to the local optima*.

Keywords- Numerical Optimization, Swirling Motion, Particle Swirl Algorithm, Vortex Particle Theory

1. INTRODUCTION

Optimization [6] is the most important process faced by every Human being during their life time. The process of optimization is the process of obtaining the „*best*", from the possible „good" or „bad" things. Optimization problems occur in most disciplines like engineering, physics, mathematics, economics, administration, commerce, social sciences, and even politics. Approaches to optimization problems are categorized as analytical, graphical, and numerical. Numerical methods [11] are the most important general approach to optimization in which iterative numerical procedures are used to generate a series of progressively improved solutions to the optimization problem, starting with an initial estimate for the solution. The process is terminated when some convergence criterion is satisfied. During the past 40 years, several branches of mathematical programming have evolved such as Linear Programming, Integer Programming, Quadratic Programming, Nonlinear Programming, and Dynamic Programming. But these branches of mathematical programming are concerned with a specific class of optimization problems.

Several modern heuristic algorithms [8] have been developed for solving optimization problems. Depending on the criteria being considered, these algorithms can be classified into iterative based, stochastic, and population based. Algorithms employing multiple iterations to find the solution are called iterative algorithm where as algorithms employing a probabilistic rule for improving a solution is called probabilistic or stochastic. Algorithms working with a set of solutions and trying to improve them are called population based algorithms.

Depending on the nature of phenomenon used, the population based algorithms further classified into Evolutionary Algorithms (EA) and Swarm Intelligence (SI) based algorithms.

The most popular EA is Genetic Algorithm (GA) [9] that attempts to simulate the phenomenon of natural evolution. The most popular SI is Particle Swarm Optimization (PSO) [7] which simulates the social behaviour of bird flocking or fishes schooling. Attempts can also be made to develop a hybrid model [12] of GA and PSO by combining their strengths but that also didn"t produce a satisfactory result for several real world problems. During the last decade several nature inspired algorithms are developed for optimization. Some of them are, Ant Colony Optimization (ACO) that simulates the forging behaviour of ants, Fish Swarm Algorithm (FSA) that simulates the praying and swarming behaviour of fishes, Invasive Weed Optimization (IWO) that simulates the colonizing behaviour of weeds, Harmony Search Algorithm (HSA) that simulates the character of music improvisation and so on.

Recently several algorithms have been developed based on the social behaviour of honey bees such as Marriage Bee Optimization (MBO) [1], Virtual Bee Algorithm (VBA) (Lucic et al., 2001), Artificial Bee Colony (ABC) [17], Bee Colony Optimization (BCO) [14], Honey Bees Mating Optimization (HBMO) [2] are the algorithms developed based on the intelligent behaviour of bees. In this paper, a novel attempt is made to simulate the motion of water in the sink. When the drain of the sink is opened, a swirling motion is started in water mass near the drain and result in release of water through the drain. Inspired from swirl motion of water particles with in a sink that search the drain, an algorithm called Particle Swirl Algorithm (PSA) is developed. The structure of rest of this paper is described as, Section 2 gives the overview of swirl motion in nature. In Section 3, details of the proposed Particle Swirl Algorithm are presented. Implementation issues of the proposed Particle Swirl Algorithm are discussed in Section 4. Details of simulations conducted using six bench mark functions and the results are reported in Section 5. Concluding remarks are given in Section 6.

2. SWIRL MOTION IN NATURE

Water is an inevitable substance in the nature for all the livings. Water has peculiar characteristics that when it is poured or contained in a sink, it tries to find the drain (hole) in the sink continuously to leave out. The answer for this excellent natural behavior of water can be found in the field of Fluid Dynamics [4] that deals with the nature of fluid flow. When the drain of the sink is opened, a swirling motion is started in the water mass near to the drain leading to the release of water though the drain. The swirl motion of water leads to an important phenomenon called vortex formation. Vortex is a spinning turbulent flow of fluid with closed streamline and it is considered as center for swirl motion of water.

Imagine a single vortex ring composed of specified number (N_φ) of disks, radius *R*=1, and decaying over 5 simulated seconds as shown in figure 1. Each disk is divided into desired number of layers (n_c) which in turn composed of a number of circumferentially distributed cells of equal area [16]. The circumferential angle between disks that compose the ring can be found as $\Delta\varphi = 2\pi / N_\varphi$. With all these definitions the cell volume is defined as

$$vol = \Delta\varphi\,(r_2 - r_1)\left[(\theta_2-\theta_1)\,R\left(\frac{r_1+r_2}{2}\right) + (\sin(\theta_2)-\sin(\theta_1))\left(\frac{r_1^2 + r_1 r_2 + r_2^2}{3}\right)\right] \quad (1)$$

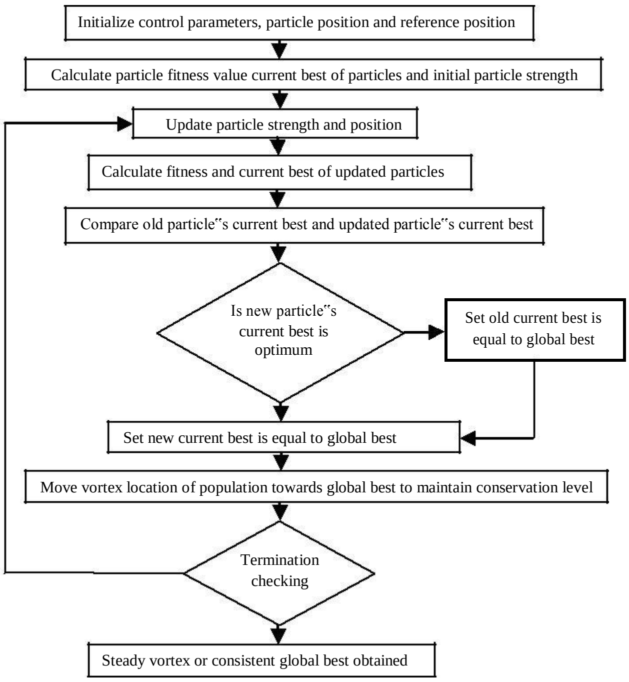

Fig 1 Flowchart of PSA

A vortex particle is an influencing element that interacts with each other and evolves in the flow field. Each vortex particle has position and strength associated with it, along with a volume and a vortex core radius. While taking part in swirl motion, particles will undergo consistent change of position with an increased tangential velocity and reducing radius towards the vortex. The strength of a particle depends on the volume and vorticity of the particle. The strength vector of the particle will also vary as the particle update its position.

3. PROPOSED OPTIMIZATION ALGORITHM

Inspired from swirl movement of water particles with in a sink that search the drain, an algorithm called Particle Swirl Algorithm (PSA) can be developed. Initially N numbers of particles are considered with best solution found so far called the global best. However the best solution calculated from the present position of particles is known as current best. The global best and the current best are consistently compared and if current best has better value than global best then it replaces the global best. The centre location of the population migrates towards the global best making gradual changes, and magnitude of such a change is called conservation level.

Fluid mechanics concepts are used to produce position update of rotating particles towards the vortex. Necessary adjustments are taken to avoid new positions of the particles going beyond the border or the range. The global best is considered as the stable vortex until no new global best is observed in a sensible amount of time or iterations. As shown in figure1 PSA parameters are initialized along with randomly generated position of particles. Fitness of particle positions is evaluated and the best one is considered as the current best. The current best is compared with an assumed global best; if the current best has better value than global best then it replaces the global best. An updating equation is applied to determine the next position of particles and the new global best is calculated. Border violations are strictly restricted and the conservation level is maintained by moving centre location of the population towards the global best value. Predetermined numbers of iterations are carried out until new global best is found.

4. IMPLEMENTATION OF PROPOSED PSA ALGORITHM

While applying the PSA algorithm for any function optimization, the following issues are to be addressed

- Initialization
- Fitness Evaluation
- Strength updating
- Position updating
- Termination Checking

4.1 Initialization of parameters and population

The parameters initialized in the PSA algorithm are Variable"s range, Number of particles (N), Maximum number of iterations (I) and Dimensions (D). The performance of the algorithm depends on the values associated with the parameters. Particle positions are randomly generated initially.

4.2 Evaluate the fitness of particles

Evaluation of the individuals in the population is accomplished by calculating the objective function value for the problem. The result of the objective function is used to calculate the fitness value of the individual. During the PSA run, it searches for a solution with maximum fitness function value. Hence the minimization objective function is transformed into maximization fitness function by

$$Fitness = k / f \qquad (2)$$

Where, k is a constant and it is to amplify $(1/f)$, the value of which is usually small, so that the fitness of the individual will be obtained in a wider range.

4.3 Particle Strength Updating

During swirling motion, the initial strength vector of the particle is updated and new strength vector is gained by the particle every time till it releases out from the drain as per the equation (1)

$$\alpha^{n+1}{}_{p,new} = \alpha_{p,ref} + \alpha^{n}{}_{p,new} - \sum_{q} vol_q \, \alpha^{n}{}_{q,new} \, \varsigma_\sigma \left(x_p - x_q \right) \qquad (3)$$

where x_p is the particle"s position, x_q is particle"s reference position, $\alpha_{p,ref}$ is strength vector of the reference particle, $\alpha^{n}{}_{p,new}$ is initial strength vector of the particle, vol_q is the cell volume given in equation (1). ς_σ is the regularization function used in vortex theory and it is given in equation (4)

$$\varsigma_\sigma (x_p - x_q) = (1/\sigma^3) \times 7.5 / 4\pi ((|x_p - x_q| / \sigma)^2 + 1)^{7/2} \qquad (4)$$

4.4 Particle Position Updating

While taking part in swirl motion particles will undergo consistent change of position with an increased tangential velocity and reducing radius towards the vortex (Winckelmans 1989),(Beale 1987). From randomly initialized positions, particles will take new positions based on an updating equation. This procedure will follow in all iterations and step by step updating will take the particles towards the vortex.

$$x_p\, update = \frac{-1}{4\Pi} \times \sum_q \frac{\left|x_p - x_q\right|^2 + \frac{5}{2}\sigma^2}{\left(\left|x_p - x_q\right|^2 + \sigma^2\right)^{\frac{5}{2}}} \left(x_p - x_q\right) \times \alpha_q \quad (5)$$

Here σ is the vortex core radius with value 0.1, x_p is the randomly initialized particle position matrix and will be updated in every iteration, x_q is the reference matrix initialized randomly, and α_q is the particle strength vector. For the updating of particles in x_p matrix, every element of matrix x_p is relatively compared with reference matrix x_q .

4.5 Termination checking

If the global best value is consistently pertaining for predetermined number iterations, then that global best value is considered as a steady vortex. For a definite set of particles, processing completion is thus reached.

5. SIMULATION RESULT

The proposed PSA approach is implemented in MATLAB and executed in a PC with Pentium i5 processor with 2.40 GHz speed and 4 GB of RAM. The proposed algorithm is run with different values of PSA control parameters and the optimal results are obtained with the following setting –

N: 30, I: 2000, Trials : 30. From figure 2, it is found that the proposed PSA widens the exploration zone and improves the convergence speed.

Table 1 Benchmark Functions

Test Function	Equation	Range
Sphere	$f(x) = \sum_{i=1}^{n} x_i^2$	[-600, 600]
Rosenbrock	$f(x) = \sum_{i=1}^{n} \left(100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right)$	[-15 15]
Rastrigin	$f(x) = \sum_{i=1}^{n} \left(x_i^2 - 10\cos(2\Pi x_i) + 10\right)$	[-15 15]
Griewank	$f(x) = (1/4000) * \sum_{i=1}^{D} (x_i)^2 - \prod_{i=1}^{D} \cos(x_i/\sqrt{i}) + 1$	[-600 600]
Ackley	$f(x) = -20\exp\left(-0.2 * \sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos 2\Pi x_i\right) + 20 + e$	[-25 25]
Schwefel	$f(x) = D * 418.9829 + \sum_{i-1}^{D} - x_i * \sin(\sqrt{\lvert x_i \rvert})$	[-500, 500]

Table 2 Performance comparison of PSA and PSO algorithms

Test Functions	Dimensions	PSO		PSA	
		Mean	Standard Deviation	Mean	Standard Deviation
Sphere	5	1.905718E+001	6.896415E+000	2.542786E-013	8.727334E-013
	10	5.275100E+001	1.124552E+001	5.212255E-011	9.351175E-012
	15	9.844528E+001	1.940551E+001	8.405856E-009	1.300960E-008
Rosenbrock	5	4.323620E+003	4.807607E+003	3.993401E+000	3.524345E-004
	10	4.256006E+004	2.193893E+004	8.979218E+000	6.799889E-006
	15	8.586910E+004	2.985230E+004	1.392814E+001	2.338330E-003
Rastrigin	5	5.728666E+001	1.055493E+001	3.024265E-008	1.649664E-007
	10	1.435107E+002	1.600838E+001	5.624723E-009	1.063408E-008
	15	2.388439E+002	1.999933E+001	1.587714E-007	8.503336E-007
Griewank	5	5.500366E-001	1.918409E-001	2.445451E-014	1.339429E-013
	10	9.823725E-001	3.866871E-002	7.378800E-008	4.041533E-007

	15	1.021461E+000	1.065359E-25	6.879541E-6	4.987321E-5
Ackley	5	7.460785E+000	1.079192E+000	8.094362E-006	4.069441E-005
	10	9.223569E+000	7.268753E-001	7.047849E-006	3.458404E-005
	15	9.712117E+000	5.614871E-001	1.007766E-004	4.978665E-004
Schwefel	5	2.903378E+001	3.653976E+000	1.029505E-004	1.404499E-005
	10	1.766015E+001	2.032029E+000	1.306130E-003	1.647748E-004
	15	7.364699E+000	1.833420E+000	1.801383E-003	3.037247E-004

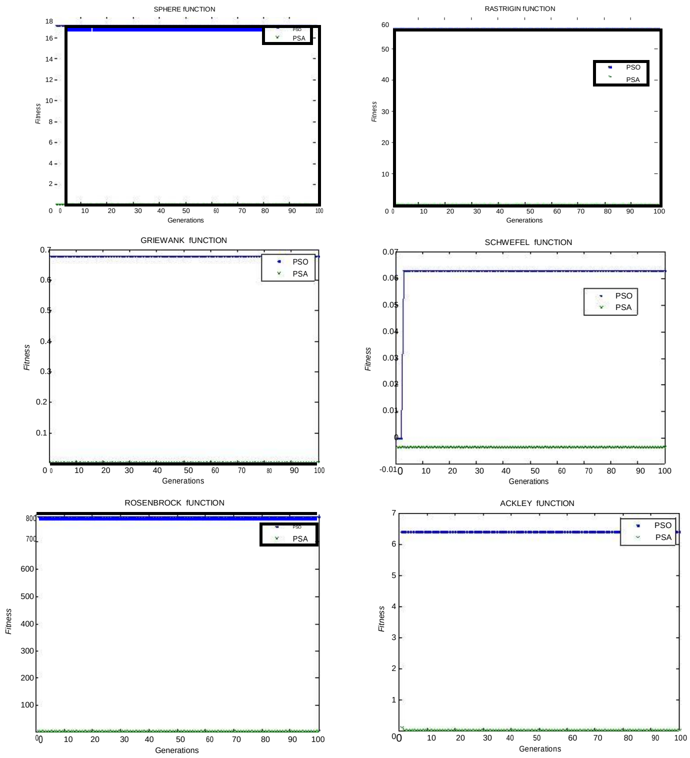

Figure 2 Convergences comparison of PSA and PSO algorithms

6. CONCLUSION

In this paper, Particle Swirl Algorithm is proposed. The proposed PSA uses the concept swirling motion of particles around a vortex that moves toward the global best position. The performance of the proposed optimization algorithm has been tested using six well-known numerical benchmark functions. From the simulation result it is found that the PSA has significant level of performance and can be efficiently applied to solve the much kind of complex optimization problems.

REFERENCES

[1] Abbass H.A., "MBO: Marriage in Honey Bees Optimization A Haplometrosis Polygynous Swarming Approach," CEC2001 Proceedings of the Congress on Evolutionary Computation, Seoul, Korea, pp.207-214, 2001.

[2] Afshar, O. Bozorg Haddad, M.A. Marino, and B.J. Adams, "Honey-Bee Mating Optimization (HBMO) Algorithm for Optimal Reservoir Operation, Modeling, Simulation and Applied Optimization," Journal of the Franklin Institute, Volume 344(5), 452- 462, 2007.

[3] Beale, J., "On The Accuracy of Vortex Methods at Large Times," Tech. rep., Institute for Mathematics and its Applications, 1987.

[4] Cengel Y., Cimbala J., "Fluid Mechanics Fundamentals andApplications" Mcgraw- Hill New York, NY 10020 2006

[5] Cottet, G.H., and Koumoutsakos, P. D., Vortex Methods: Theory and Practice, Cambridge University Press, 2000.

[6] Jorge Nocedal Stephen Wright J., "Numerical optimization" Second Edition ,Springer Series in Operation Research and Financial Engineering, 2006.

[7] Konstantinos E. Parsopoulos and Michael N. Vrahatis "On the Computation of All Global Minimizers Through Particle Swarm Optimization", IEEE Transactions On Evolutionary Computation, Vol. 8, No. 3, June 2004

[8] Pham, D. T., Karaboga, D. : Intelligent Optimization Techniques. Springer, London, 2000.

[9] Randy L. Haupt, S. E. Haupt "Practical genetic algorithms" Wiley-IEEE, 2004

[10] Seeley T.D. and S.C. Buhrman, "Group Decision Making in Swarms of Honey Bees," Behav. Ecol. Socio biol., 1999.

[11] Singiresu S Rao " Engineering Optimization: Theory And Practice" John Wily & sons, 1996.

[12] Srinivasan, D., Seow, T.H "Evolutionary Computation", CEC "03, 8-12 Dec. 2003, Canberre, Australia, pp. 2292-2297

[13] Storn R., and Price K. V., "Differential Evolution–A simple and efficient adaptive scheme for global optimization over continuous spaces," Institute of Company Secretaries of India, Chennai, Tamil Nadu. Tech. Report TR-95-012, 1995.

[14] Teodorovic D., and M.Dell"Orco, "Bee Colony Optimization- A Cooperative Learning Approach to Complex Transportation Problems," Advanced OR and AI Methods in Transportation, pp.51-60,2005.

[15] Wedde, H.F., Farooq, M., Zhang, Y "Bee Hive: An efficient fault-tolerant routing algorithm inspired by honey bee behaviour, ant colony, optimization and swarm intelligence.", Proceedings of the 4th International Workshop, ANTS 2004, Brussels, Belgium, 5–8 September 2004.

[16] Winckelmans, G. S., Topics in Vortex Methods for Computation of Three- and Two-Dimensional Incompressible Unsteady Flows, Doctor of philosophy, California Institute of Technology, Pasadena, California, February 1989.

[17] Yang X.S., "Engineering Optimizations via Nature-Inspired Virtual Bee Algorithms," IWINAC 2005, Springer-Verlag, Berlin Heidelberg, pp. 317-323, 2005.